Newton's Abductive Methodology[*]

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Abstract

[11] The Newtonian programme consists of the Principia's core axioms, a set of force laws, and a bundle of methodological rules. The latter underwent several changes for which reason it is claimed that Newton and the Newtonians added methodological rules *post constructione* in order to further support their research agenda. In particular, Duhem, Feyerabend, and Lakatos aimed to provide a theoretical reason why Newton could not have come up with his theory in accordance with his own abductive methodology. In this paper, Newton's method is characterised and general background assumptions of Duhem et al.'s argument are made explicit. Subsequently, the argument is criticised based on a contemporary philosophy of science point of view.

Keywords: Principia Mathematica, Newton's method, deduction from the phenomena, induction, overfitting

Sir Isaac Newton is considered to be the flagbearer of promoting the *inductive method*. With his *Principia* he put forward a framework which marked the route of physics for more than two centuries both in theoretical as well as methodological respect. However, *modern philosophy of science theorising* about Newton's methodology and actual procedure states that he did not "practice what he preached". This contribution is about Newton's "*walk the talk*". Our investigation proceeds as follows: In section 1, we outline Newton's methodology; in section 2, we describe the critique of his actual procedure; finally, in section 3, we undermine the critique and provide a brief rational reconstruction of his approach.

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1 Newton's Methodology

The sources of Newton's methodology are:

- *Philosophiæ Naturalis Principia Mathematica*, particularly:
  - Preface to the first edition (cf. Newton 1726(E3)/1999, pp.381–383)
  - Cotes' preface to the second edition (cf. Newton 1726(E3)/1999, pp.385–399)
  - *Scholium* in Book 1, Section 11 (cf. Smith 2002, p.140; and Newton 1726(E3)/1999, pp.588f)
  - *Regulæ Philosophandi* (Rules of natural science, cf. Newton 1726(E3)/1999, pp.794–796 (Book 3))
  - *Scholium Generale* (cf. Newton 1726(E3)/1999, pp.939–945 (Book 3))
- *Opticks: Or, a Treatise of the Reflections, Refractions, Inflections and Colours of Light* (cf. Newton 1721), particularly:
  - Query 31 (cf. Newton 1721, p.380 (Book 3))
- Correspondence, particularly:
  - Correspondence with Henry Oldenburg (cf. Lakatos 1980, p.218; and Feyerabend 1978, p.206)

Newton's methodology is embedded into the Newtonian research programme. In the terminology of Imre Lakatos' *scientific research programmes*, the Newtonian programme of the *Principia* consists of a *core*, which is in Newton's case the set of three general axioms on forces:

> *Law 1*: "Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed":
>
> $$\forall x, t : \sum_{i \in S} f_i(x,t) = 0 \Rightarrow a(x,t) = 0$$
>
> *Law 2*: "A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed":
>
> $$\forall x, t : \sum_{i \in S} f_i(x,t) = m(x) \cdot a(x,t)$$
>
> *Law 3*: "To any action there is always an opposite and equal reaction;":
>
> $$\forall x, t \forall i \in S \exists j \in S : f_i(x,t) = -f_j(x,t)$$

and a *periphery*, namely further axioms about specific forces etc.:

> *Law G*: "Gravity exists in all bodies universally and is proportional to the quantity of matter in each. [...] The gravitation toward each [...] body is inversely as the square of the distance of places from those [bodies].":
>
> $$\forall x, y, t : f_g(x,y,t) = G \cdot \frac{m(x) \cdot m(y)}{d(x,y,t)^2}$$

The methodological part of the programme was stepwise expanded:

- *E1*: First Edition of the *Principia* (1687, published with support by Edmond Halley)

- *E2*: Second Edition of the *Principia* (1713, edited by Roger Cotes)
- *E3*: Third Edition of the *Principia* (1726, edited by Henry Pemberton)

In his writings, Newton proposes 5 *methodological rules* (*regulæ*). They appear in the *Principia* starting with edition E2 and E3; more specifically, *regulæ I–III* appear in E2; and in E3 *regula IV* is added; and a further rule, *regula V*, appears only in the manuscript of E3, but not in print. Newton's so-called *regulæ philosophandi* are the following (cf. Newton 1726(E3)/1999, pp.794–796):

I "*No more causes* of natural things should be admitted *than* are both true and *sufficient* to explain their phenomena."

II "Therefore, the *causes* assigned to natural *effects of the same kind* must be, so far as possible, *the same*."

III "Those qualities of bodies that cannot be intended and remitted [i.e., qualities that cannot be increased and diminished] and that belong to all bodies on which experiments can be made should be taken as qualities of all bodies universally."

IV "In experimental philosophy, *propositions gathered from phenomena by induction should be considered either exactly or very nearly true notwithstanding any contrary hypotheses*, until yet other phenomena make such propositions either more exact or liable to exceptions."

The last rule, which is present only in the manuscript of E3, is (cf. Koyré 1965, p.272):

V "Whatever is not derived from things themselves, whether by the external senses or by the sensation of internal thoughts, is to be taken for a hypothesis. [...] *And those things which neither can be demonstrated from the phenomenon nor follow from it by the argument of induction, I hold as hypotheses*."

[12] Regarding the interpretation of the *regulæ*, one can observe that *regulæ* I–II are usually interpreted as parsimony principles. Their epistemic rationale can be characterised as follows (cf. Forster and Sober 1994, sect.4): Assume X is to be explained causally (by help of probabilities Pr):

$Pr(X\|\cdot)$	C_1	$\sim C_1$
C_2	$c_0, c_1, c_2, c_{1,2}$	c_0, c_2
$\sim C_2$	c_0, c_1	c_0

Here the c_is are the parameters of the models. Then there are several options for providing such a causal explanation:

1. $Pr(X|C_1, C_2) = c_0 + c_1 \cdot val(C_1)$ (*single cause*)
2. $Pr(X|C_1, C_2) = c_0 + c_1 \cdot val(C_1) + c_2 \cdot val(C_2)$ (*non-interactive causes*)
3. $Pr(X|C_1, C_2) = c_0 + c_1 \cdot val(C_1) + c_2 \cdot val(C_2) + i_{c_1,c_2} \cdot val(C_1) \cdot val(C_2)$ (*interactive causes*)

By employing reasoning of model selection we get:

- *Accuracy* (better in 3>2>1)
  ⇒ on average more fitting of errors in data, i.e. *overfitting*.

- *Simplicity* (better in 1>2>3)
  ⇒ on average less fitting of errors in data.

According to celebrated arguments of model selection, one needs to *balance accuracy and simplicity* in order to maximise one's expected accuracy. Hence, one can state in line with Newton that in case of $\text{accuracy}_1 = \text{accuracy}_2$ one should opt for the simpler model. Hence, the epistemic value of simplicity.

Regarding the interpretation of *regulæ* IV–V, we can observe that they have *programmatic* value only: It is clear that particularly *regula* V is directed against Cartesians (vortex theory). These rules are intended to license inductive inference and shield it against *a priori* rationalistic theorising (cf. also Newton's: "*Hypotheses non fingo*"). In general, one might wonder why *regula* V was not included in the printed version of E3. With regards to this, Koyré (1965, p.272) conjectures that Newton and the Newtonians did not want to include the anti-Cartesian polemic in the scientific text itself.

Now, since *induction* is so prominently stated in this rules, we have to clarify its role in Newton's methodology, and for this we have to delve a little bit into *Newton's method*. Newton's method is also called the method of *analysis and synthesis* (cf. Duhem 1954, p.190). Newton himself wrote about it:

> "The basic problem of philosophy seems to be to *discover the forces of nature from the phenomena* of motions and *then to demonstrate the other phenomena from these forces*." (Newton 1726(E3)/1999, Preface to the first edition, p.382)
> and:
> "*Analysis consists in* making Experiments and Observations, and in *drawing general Conclusions from them by Induction*, and admitting of no Objections against the Conclusions, but such as are taken from Experiments, or other certain Truths. For Hypotheses are not to be regarded in experimental Philosophy. [...] By this way of Analysis we may *proceed from [...] Effects to their Causes*. [...] The *Synthesis consists in* assuming the Causes discover'd, and establish'd as Principles, and by them *explaining the Phaenomena proceeding from them*, and proving the Explanations." (cf. Newton 1721, Query 31, p.380)

The method differs from hypothetico-deductivism in the sense that according to it *not any hypothesis* is admissible. The schema of the method is according to Hintikka and Remes (1974, p.110) as follows:

[*Analysis:*]

i an analysis of a certain situation into its ingredients and factors
→

ii an examination of the interdependencies between these factors
→

iii a generalization of the relationships so discovered to all similar situations
→

[*Synthesis:*]

iv deductive applications of these general laws to explain and to predict other situations.

Induction in a narrow sense occurs only as step (iii). Step (i) and (ii) are *abductive* ones. This abductive part concerns reasoning from the "effect to the causes"; in modern approaches it is dealt within theories of *common cause abduction*. The rough idea of such an approach is as follows (cf. Feldbacher-Escamilla and Gebharter 2019; Schurz 2008):

- A *correlation among empirical phenomena* is observed.
- Probabilistically speaking, this correlation can be explained by assuming a *common cause*.
- This kind of explanation is also to be preferred against other kinds of explanation (with different structures: common effect, common cause with intermediate causes, etc.), because it allows for *unification*.
- Unification can be epistemically justified again by the aim of *avoiding overfitting* (cf. Forster and Sober 1994, sect.3).

The different taxonomies of elements of *Newton's method* and their relations in the literature are as follows:

Newton	*Analysis*			*Synthesis*
	Decomposition			*Recomposition*
	Regulæ I & *II*		*Regulæ III* & *IV*	logic, geometry, *calculus*, etc.
Hintikka et al.	factor analysis (i)	dependency analysis (ii)	generalisation (iii)	deductive application (iv)
Duhem et al.	induction in the wide sense			deductive application
Example	Kepler's laws $\Rightarrow$ Newton's laws (e.g. law of gravitation)			Newton's laws $\Rightarrow$ description of the moon's orbit
Modern	general: data analysis with methodological norm: parsimony (cf. Forster and Sober 1994)	e.g. Bayes net analysis with causal relations (cf. Pearl 2000)	e.g. *inductive generalisation in the narrow sense* ("inductive logic")	*deductive methods* (logic, mathematics)
	abduction			

It is important to note that according to Duhem (1954), the inductive part consists in components (i–iii), and that in this part Newton proceeded from Kepler's laws to his law of gravitation.

So much for what Newton "preached" as proper scientific methodology. Now let us come to Duhem et al.'s critique of Newton's scientific practice.

2 Critique on Newton's Actual Procedure

A *rational reconstruction* of Newton's theorising and his methodological suggestions led to sharp *critique* in the 20th century. Particularly Duhem (1954) put forward an inconsistency objection, which was later on continued by Lakatos (1980, p.213), Paul Feyerabend (1981, p.174, 175, 206), and also Karl R. Popper (1983, p.140). According to them, Newton did not comply with his own standards:

> "The *principle of universal gravity, very far from being derivable by generalization and induction from the observational laws of Kepler, formally contradicts these laws*. If Newton's theory is correct, Kepler's laws are necessarily false. (cf. Duhem 1954, p.193)

Their argument can be summarised as follows:

1. Kepler's laws are, conditional on the—at Newton's time—accepted auxiliary assumptions, in contradiction to the observed orbits of the planets.
2. Newton's theory of motion in the solar system was and is quite accurate.
3. Hence: Kepler's and Newton's theories are incompatible. (from 1, 2)
4. [13] Hence: Newton could not came up with his theory *inductively* on the basis of Kepler's laws. (from 3)

So, according to the mentioned authors, Newton did not comply with his own rules, and he did so, they add, because he wanted to push his research programme. And, indeed, in studying the different editions (E1,E2,E3) one sees that he added the *regulæ post constructione* which might have in fact happened in order to *"push" his programme*.

However, here we are not after an historical evaluation of Newton's and the Newtonian's motives. Rather, we want to investigate whether, for systematic reasons, one can really blame them to fail their own methodological constraints. And as we will see, one cannot.

3 Rationalisation of Newton's Actual Procedure

As we have outlined Newton's methodology in section 1 and presented a main methodological criticism against Newton in section 2, we will argue now that Newton's actual procedure can be considered to be rational and that the provided theoretical reason against it (namely that what he had inductively gained from his basis, contradicts the basis) is inconclusive.

First, note that Duhem et al.'s argument as it stands is not deductively valid. An important assumption to validate the argument is another premiss which links inductive and deductive reasoning in the following way:

> **Principle on the Relation Induction-Deduction (PRID)**
> If H is inductively inferred from E, then E can be deduced from H (plus auxiliary assumptions) *or* E is at least not incompatible with H.

Given (PRID), we can validate the inferential step from 3. to 4. in the argument above. However, as we will show now, Newton's actual procedure was more fine-grained, and the fine-grained procedure satisfies (PRID). Furthermore, if it were not to satisfy (PRID), this would not pose much a problem, because (PRID) itself does not stand up to scrutiny.

Let us come to the first objection. An important approach in the literature reconciles Newton's theorising with his methodological proposal by *restricting the basis of analysis*. According to this approach, in adequately describing

Newton's actual procedure, the basis, i.e. *Kepler's laws*, has to be restricted to instances which are compatible with the result of analysis, namely Newton's theory. This argument is put forward, e.g., in (cf. Smith 2002; Ducheyne 2012):

> "On closer scrutiny, the so-called *contradiction [...] is simply non-existent* – as any reader of the first three propositions of Book I and Phenomena I–VI as stated in Book III of the *Principia* can testify. The particular criticism raised is beside the point, as *Newton demonstrated that exact Keplerian motion occurs only in one-body systems* and that, under specific configurations, Keplerian motion occurs as most closely as possible (*quam proxime*) in three- and many-body systems as well. [...] There is no formal contradiction involved whatsoever." (cf. Ducheyne 2012, p.XV)

Now, here is a schema how one can resolve inconsistency by help of a restricted domain of *analysis*:

- *Basis*: three laws of Kepler: X
- *Restriction for analysis to one-body systems*: X'
- *Analysis*, applied to X', with inductive generalisation and the results of the first book of the *Principia*: Y
- *Synthesis*: Deduction of X' and further phenomena from Y (cf. phenomenon 4 in Newton 1726(E3)/1999, p.800)

Since *in the restricted domain* (one-body systems) X' is compatible with Y—X' is even a consequence of Y—, the principle on the relation between induction and deduction (PRID) is satisfied.

Let us now come to the other route of resolving inconsistency, namely to *argue against (PRID)*. For this, first, observe that (PRID) has two readings:

R1 $E \mid\!\sim H \;\Rightarrow\; H \vdash E$ (where $\mid\!\sim$ stands for an inductive inference)

R2 $E \mid\!\sim H \;\Rightarrow\; H \nvdash \neg E$

Note that common confirmation theory disproves R1, because it licenses $E \mid\!\sim H$ via high enough $conf(H, E)$ in many cases where $H \nvdash E$. However, in order to validate the argument of Duhem et al. against Newton, one only needs the weaker reading R2 (which follows from R1 in case that H is consistent). However, even the weaker reading R2 is invalid. One can show this, by again employing the *overfitting*-argument from model selection. Here is the *overfitting*-argument against (PRID):

1. Assume: $E \mid\!\sim H$ and H to be consistent
2. Now, since $E \mid\!\sim H$, H will not be undetermined regarding E, because in model selection E or $\neg E$ is always covered by H in such a case. (model selection, 1)
3. So, either $H \vdash E$ or $H \vdash \neg E$. (from 2)
4. But then it follows that $H \vdash E$. (with 1, 3, R2 of (PRID))
5. But this amounts to demand that H perfectly fits E. (model selection, 4)
6. But then H is prone to *overfit* E, i.e. fit errors in E. (model selection, 5)
7. However, H should not be prone to *overfit* E. (basic epistemic desideratum)

8. Hence, (PRID) should be abandoned. (from 1–7)

Let us briefly sum up. We have outlined the Newtonian *research programme*: its *theoretical* part consists of the core axioms of the *Principia* and its *methodological* part of the *regulæ philosophandi*. We have seen that the methodology (Newton's method) relevantly contains abduction (*analysis*), induction (*analysis*), and deduction (*synthesis*). We presented a common argument on *practical inconsistency* of Newton's actual procedure, namely that the basis for the inductive part of analysis (Kepler's laws) is inconsistent with its result (Newton's theory). We have identified a background assumption of this argument, namely the principle on the relation induction-deduction (PRID), and we have argued against (PRID) by help of *overfitting*-considerations. [14]

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