Language Dependence Redeflated

Christian J. Feldbacher-Escamilla

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Introduction

We are familiar with evaluations of the form:

- $\mathcal{T} \vdash mp = 5.74''/a$
- $N \vdash mp = 5.74''/a - 0.4311''/a$
- $R \vdash mp = 5.74''/a - 0.08''/a$
- So R is better than N, R is more confirmed by the facts than N, R is more accurate than N, R has more verisimilitude than N etc.

Within usual approaches to confirmation, accuracy, verisimilitude etc. one tries to define logical concepts that allow an evaluation of this kind.

But all of the common definitions seem to depend on the language the compared theories are built of.

Contents

The Problem of Language Dependence

Popper's theory of verisimilitude

Karl R. Popper's quantitative theory of verisimilitude:

Definition (cf. (Popper 1972), pp.334ff)

- Truth content: $ct_{T,\mathcal{T}}(X) = 1 - p(X_{T,\mathcal{T}})$
- Falsity content: $ct_{F,\mathcal{T}}(X) = 1 - p(X|X_{T,\mathcal{T}})$
- A measure: $vs_{\mathcal{T}}(X) = ct_{T,\mathcal{T}}(X) - ct_{F,\mathcal{T}}(X)$

Problem:

Theorem (cf. (Tichý 1974), p.158)

For all X, Y and $\mathcal{T}$: if $X_{F,\mathcal{T}} \neq \emptyset$, $Y_{F,\mathcal{T}} \neq \emptyset$ and $p(X) = p(Y)$, then $vs_{\mathcal{T}}(X) = vs_{\mathcal{T}}(Y)$.

Tichý's critique of the probabilistic account

Pavel Tichý's quantitative theory of (sheer counting) verisimilitude:

Definition (cf. (Tichý 1976) and (Miller 1976))

- Constituents: $m(A) = |\{X : \textit{there is a } n \textit{ such that } X = \langle n, \vee \rangle \in dnf(A)\}| + 1$
- Atomic errors: $k_{\mathcal{T}}(A) = \sum_{i=0}^{m(A)-1} |\{X : \textit{there is a } n, y \textit{ such that } X = \langle n+i, y \rangle \in dnf(A) \textit{ and } \langle n\%|\{Z : \textit{there is a } o \textit{ such that } Z = \langle o, \& \rangle \in dnf(\mathcal{T})\}| + 1, y \rangle \notin dnf(\mathcal{T})\}|$
- A measure: $\Delta_{\mathcal{T}}(A) = \frac{k_{\mathcal{T}}(A)}{m(A)}$

Tichý's critique of the probabilistic account

An example:

X	$dnf(X)$	$m(X)$	$k_{\mathcal{T}}(X)$	$\Delta_{\mathcal{T}}(X)$
$\mathcal{T}$	h & r & w	1	0	0
A	$\overline{h}$ & r & w	1	1	1
B	$\overline{h}$ & $\overline{r}$ & $\overline{w}$	1	3	3
$\vdots$				

Note that:

- $A_{F,\mathcal{T}} \neq \emptyset$, $B_{F,\mathcal{T}} \neq \emptyset$
- $p(A) = p(B)$ (constituents are mutually incompatible and $\mathcal{T}, A, B, \ldots, G$ are jointly exhaustive)
- And hence: $vs_{\mathcal{T}}(A) = vs_{\mathcal{T}}(B)$
- Although: $\Delta_{\mathcal{T}}(A) < \Delta_{\mathcal{T}}(B)$
- And that's one reason why Popper's quantitative theory of verisimilitude fails.

David Miller's critique of Tichý's critique

Take another language (cf. Miller 1974, pp.175ff):

- $m \leftrightarrow (h \leftrightarrow r)$
- $a \leftrightarrow (h \leftrightarrow w)$

X	$dnf(X)$	$m(X)$	$k_{\mathcal{T}'}(X)$	$\Delta_{\mathcal{T}'}(X)$
$\mathcal{T}'$	$h \,\&\, m \,\&\, a$	1	0	0
A'	$\overline{h} \,\&\, \overline{m} \,\&\, \overline{a}$	1	3	3
B'	$\overline{h} \,\&\, m \,\&\, a$	1	1	1

Note that:

- $\Delta_{\mathcal{T}}(B') < \Delta_{\mathcal{T}}(A')$
- Although: $\Delta_{\mathcal{T}}(A) < \Delta_{\mathcal{T}}(B)$ and A, A' and B, B' are synonymous or equivalent theories inasmuch as they are intertranslatable:
  - $r \leftrightarrow (h \leftrightarrow m)$
  - $w \leftrightarrow (h \leftrightarrow w)$

So the ranking of theories depends on the language you choose for formulating the theories.

The problem of language dependence

Definition (cf. (Popper 1967), p.12 and (Kanger 1968))

Two theories X and Y are synonymous resp. equivalent resp. have a common definitional extension iff there are X', Y', D_1 and D_2 such that:

- $X' = \mathit{even}(X)$ and $Y' = \mathit{odd}(Y)$ (where *even* and *odd* separate the vocabulary of X and Y)
- D_1 is a set of definitions of each non-logical expression of Y' in terms of expressions of X'
- D_2 is a set of definitions of each non-logical expression of X' in terms of expressions of Y'
- $Cn(X' \cup D_1) = Cn(Y' \cup D_2)$
- ($Cn(D_1) = Cn(D_2)$ ... needed, although not mentioned in (Kanger 1968))

Example: The elementary theories of $<$ and $\leq$ are equivalent.

The problem of language dependence

Language dependence of relations:

Definition

A n-ary relation R^n of a theory T is language dependent iff there are $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ such that

- x_1, y_1 and ... and x_n, y_n are synonymous (whereby it is assumed that there is an overall definitional extension for all of them)
- $T \vdash R^n(x_1, \ldots, x_n)$
- $T \vdash \overline{R^n(y_1, \ldots, y_n)}$

Example:
A preorder of theories by Δ of Tichý's theory is language dependent.
etc.

A Critique of the Synonymy Concept

Solutions to the problem

The ingredients of the problems put by Miller:

- A definition of synonymy amongst theories
- Unrestricted language construction
- Language dependence of many logical and methodological concepts

Offered solutions to the problem:

1. Restriction of the synonymy definition (cf. Tichý 1978)
2. Restriction of language construction (cf. Schurz 1997), (cf. Zwart 1995)
3. Acceptance of relativism (cf. Barnes 1991)

Miller accepts none of the solutions of this kind presented up to now (cf. Miller 2006, chpt.11). In the following we will give some arguments among the line of 1.

First argument

Probability preservation:

1. If X and Y are synonymous resp. equivalent, then $p(X) = p(Y)$.
2. According to Miller's applied definition of 'synonymy' it is possible that X and Y are synonymous, but $p(X) \neq p(Y)$.
3. Hence, Miller's definition is too wide.

Take the weather example subjectively interpreted by an agent α (for a more general discussion cf., e.g. (Niiniluoto 1987, chpt.13.2)):

- $A \; = \overline{h} \;\&\; r \;\&\; w$
- $A' = \overline{h} \;\&\; \overline{m} \;\&\; \overline{a}$
- A and A' are synonymous
- Since the definitions of the definitional extension are only expressions of the meta-language, it is possible that: $p_\alpha(A) \neq p_\alpha(A')$.

Second argument

Miller's applied definition of 'synonymy' is – as can be shown by some examples – too wide.

Take, e.g., the weather language of Miller. Then it's easy to show that, e.g., all A with $m(A) = 1$ are pairwise inter- and intratranslatable (i.e.: translatable within the same language) and by this synonymous (cf. Schramm 1979):

$h \;\&\; r \;\&\; w$	$h \;\&\; m \;\&\; a$
$h \;\&\; r \;\&\; \overline{w}$	$h \;\&\; m \;\&\; \overline{a}$
$h \;\&\; \overline{r} \;\&\; w$	$h \;\&\; \overline{m} \;\&\; a$
$h \;\&\; \overline{r} \;\&\; \overline{w}$	$h \;\&\; \overline{m} \;\&\; \overline{a}$
$\overline{h} \;\&\; r \;\&\; w$	$\overline{h} \;\&\; m \;\&\; a$
$\overline{h} \;\&\; r \;\&\; \overline{w}$	$\overline{h} \;\&\; m \;\&\; \overline{a}$
$\overline{h} \;\&\; \overline{r} \;\&\; w$	$\overline{h} \;\&\; \overline{m} \;\&\; a$
$\overline{h} \;\&\; \overline{r} \;\&\; \overline{w}$	$\overline{h} \;\&\; \overline{m} \;\&\; \overline{a}$

But neither $Cn(\{h \;\&\; r \;\&\; w\}), Cn(\{\overline{h} \;\&\; \overline{r} \;\&\; \overline{w}\})$ nor $Cn(\{h \;\&\; r \;\&\; w\}), Cn(\{\overline{h} \;\&\; \overline{m} \;\&\; \overline{a}\})$ seem to be intuit. synonymous.

Hence: Miller's definition of 'synonymy' is too wide.

Third argument

Miller's adequacy condition is by itself language dependent. Take the following example (where c, d and c', d' are synonymous theories):

- T: $c \neq d$ & $\forall x P(x)$
- T': $c' \neq d'$ & $\forall x P'(x) \leftrightarrow x \neq c'$
- Def.: $P(x) \leftrightarrow ((x = c' \rightarrow \neg P'(x))$ & $(x \neq c' \rightarrow P'(x)))$
- Def.: $P'(x) \leftrightarrow ((x = c \rightarrow \neg P(x))$ & $(x \neq c \rightarrow P(x)))$
- Def.: $c = c'$, $d = d'$

Then it follows (you may interpret P as 'incomparable with the truth'):

- P of T (naïve sceptic) is not language dependent.
- P' of T' (reflecting sceptic) is language dependent.

Although T and T' are synonymous.

Hence: '... of ... is language dependent' of Miller's theory (resp. adequacy condition) is language dependent.

A restricted definition

The main problem seems to be:

- Given $P_1(x)$ (representing, e.g., 'It's raining on x.') and
- $P_2(x)$ (representing, e.g., 'The Dow Jones index increases on x.'),
- no one would accept $P_1(x) \leftrightarrow P_2(x)$ as definition, i.e.: we wouldn't state $P_1(x) \leftrightarrow P_2(x)$ to be true by definition.

A natural modification of Miller's definition seems to be:

Definition

Two theories X and Y are synonymous w.r.t. an Interpretation I iff there are X', Y', D_1 and D_2 such that: $I(D_1) = I(D_2) = 1$ and ...

A restricted definition

One may define also an absolute concept of synonymity:

Definition

Two theories X and Y are synonymous iff there are X', Y', D_1, D_2 such that for all I: $I(D_1) = I(D_2) = 1$ and ...

But this concept coincides with logical equivalence:

Theorem

X and Y are synonymous iff $X \vdash\dashv Y$.

And of course all common theories of confirmation, verisimilitude etc. preserve their ordering of theories under logical translation.

(And thereby satisfy, e.g., the principle of probability preservation amongst synonymous theories.)

References I

Barnes, Eric (1991). "Beyond verisimilitude: A linguistically invariant basis for scientific progress". In: *Synthese* 88.3, pp. 309–339. URL: http://www.springerlink.com/content/n415u420641701m8/fulltext.pdf.

Feldbacher-Escamilla, Christian J. (under revision). *Epistemic Engineering. Uncovering the Logic of Deceivability and Meta-Induction*. book manuscript.

Kanger, Stig (1968). "Equivalent Theories". In: *Theoria* 34, pp. 1–6. DOI: 10.1111/j.1755;%20--2567.1968.

Miller, David (1974). "Popper's Qualitative Theory of Verisimilitude". In: *The British Journal for the Philosophy of Science* 25.2, pp. 166–177.

— (1976). "Verisimilitude Redeflated". In: *The British Journal for the Philosophy of Science* 27.4, pp. 363–381. URL: http://www.jstor.org/stable/686863.

— (2006). *Out of error: Further essays on critical rationalism*. Hampshire: Ashgate Publishing.

Niiniluoto, Ilkka (1987). *Truthlikeness*. illustrated. Synthese library (Vol 185). Berlin: Springer.

Popper, Karl R. (1967). "Quantum Mechanics without "The Observer"". In: *Quantum Theory and Reality*. Ed. by Bunge, Mario. New York: Springer, pp. 7–44.

— (1972). *Objective Knowledge*. Oxford: Oxford University Press.

Schramm, Alfons (1979). "Von der Voreiligkeit des Erkenntnisrelativismus". In: *Wittgenstein, The Vienna Circle and Critical Rationalism. Proceedings of International Wittgenstein Symposium*. Ed. by Berghel, Hal and Hübner, Adolf. Wien: Hölder-Pichler-Tempsky, pp. 283–287.

References II

Schurz, Gerhard (1997). "Sprachabhängigkeit der Erkenntnis". In: *IPS-Reprints*. Ed. by Schurz, Gerhard and Hieke, Alexander. Salzburg: Institut für Philosophie der Universität Salzburg, pp. 2–32.

Tichý, Pavel (1974). "On Popper's Definitions of Verisimilitude". In: *The British Journal for the Philosophy of Science* 25.2, pp. 155–160. DOI: 10.1093/bjps/25.2.155.

— (1976). "Verisimilitude Redefined". In: *The British Journal for the Philosophy of Science* 27.1, pp. 25–42. DOI: 10.1093/bjps/27.1.25.

— (1978). "Verisimilitude Revisited". In: *Synthese* 38.2, pp. 175–196. URL: http://www.jstor.org/stable/20115289.

Zwart, Sjoerd (1995). "A Hidden Variable in the Discussion About Language Dependency of Truthlikeness". In: *Cognitive patterns in science and common sense*. Ed. by Kuipers, Theo. Amsterdam: Rodopi, pp. 249–272.