Is Mereology Ontologically Innocent?

Well, It Depends...

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Winter 2012

Project Information

Publication(s):

- Feldbacher-Escamilla, Christian J. (2019-04). "Is Mereology Ontologically Innocent? Well, it Depends...". In: *Philosophia* 47.2, pp. 395–424. DOI: 10.1007/s11406-018-9985-6.

Talk(s):

- Feldbacher-Escamilla, Christian J. (2012a-11-20/2012-11-21). *Is Mereology Ontologically Innocent? Well, it Depends...* Conference. Presentation (contributed). International Conference of the Italian Society for Logic and Philosophy of Science (SILFS). University of Milan-Bicocca: SILFS.
- Feldbacher-Escamilla, Christian J. (2012b-03-15/2012-03-16). *The Logical Status of Mereology*. Conference. Presentation (contributed). 9th The Character of the Current Philosophy and its Methods. Slovak Academy of Sciences: Slovak Academy of Sciences.

Grant(s):

- John Templeton Foundation Grant #15571, *Analytic Theology*.

Introduction

Mereology is by no means taken to be a competitor of set theory. But for some problems it seems to be a more adequate frame.

One criteria of adequacy is that of ontological simplicity.

Of course mereology is more ontologically innocent than set theory inasmuch as it can be constructed out of set theory but not vice versa.

Nevertheless the question arises if mereology is fully ontologically innocent (and thereby a logical theory)?

Contents

Ontological Innocence

Unrestricted Composition

There are different degrees of abstraction and distinction needed in science:

One may distinguish:

- molecules from atoms
- atoms from protons etc.
- protons from Quarks etc.

And one may gather together:

- legs and boards to desks
- desks and chairs to furniture
- furniture and machines to inventory

Unrestricted Composition

Some people regard some kinds of distinction as too subtle (e.g. modern nominalists: Quine and Goodman, (Goodman 1956)), some people think that some kinds of abstraction are too generalized (e.g. opponents of unrestricted composition).

For doing good science one needs the right balance.

And for being balanced one needs the right theoretical frame:

- Set theory
- Mereology

$\vdots$

Some discussed problems regarding this matter:
$Z = X \cup Y$, $x \neq \{x\}$, $z = sum(x, y)$
A Leibnizian remark (*calculemus*): the later two problems contain many problems of mathematics and of most of the formal sciences.

Unrestricted Composition

The mereological problem of $z = sum(x, y)$ for any x and y:

- $table = sum(leg, board)$ ☑
- $board = sum(layer_1, layer_2)$ ☑
- $layer_1 = sum(wood, glue)$ ☑

⋮

- $? = sum(table, blackboard)$ ☒
- $? = sum(table, cathedral)$ ☒

⋮

Some people think that one has to restrict composition (*sum*).

Others believe that we are right balanced by using composition (cf. Lewis 1991).

Unrestricted Composition

Arguments for unrestricted composition:

- Conventionalists: individuation is a matter of convention (cf. Goodman 1956).
  Just baptise the child ($sum(table, cathedral)$)!
- Guilt-Rejectors: mereology is ontologically innocent (cf. Lewis 1991).
  There is no new entity.
  $sum(table, cathedral)$ is just your entities *table* and *cathedral*.
  So, if you accept that tables and the cathedral of Milan exist, you also accept that their composition exists.

We focus on the innocence thesis. To argue in favour of unrestricted composition, one may argue in favour of the innocence thesis and that is first of all to make sense of it.

Unrestricted Composition

Before we characterise the innocence thesis in detail, we first motivate our later characterisation. There are two main approaches:

- Identification Approach:
  "A whole is identical with its parts." (Baxter 1988b, p.197)

  "The 'are' of composition is, so to speak, the plural form of the 'is' of identity." (Lewis 1991, p.82)

- Counting Approach:
  "The whole is the many parts counted as one thing." (cf. Baxter 1988a, p.578) and (cf. Lewis 1991, p.83)

  "If you draw up an inventory of reality [. . .], it would be double counting to list the fusion of the xs and also list the xs." (cf. Lewis 1991, p.81)

We will concentrate our investigation on the identification approach only (but of course both approaches are closely connected).

Unrestricted Composition: Identification

An argument against the identification approach (cf. Yi 1999), (cf. Inwagen 1994):

1. Assume $a \neq b$ and $c = sum(a, b)$.
2. According to defenders of the innocence thesis it holds that the composition is identical with its parts, so assume $c = a$ and $c = b$.
3. The relation of being an improper part ($\preceq$) is reflexive, so it holds $c \preceq c$.
4. As far as a is different from b (1), the composition of a and b, that is c, is neither an improper part of a nor of b: $c \not\preceq a$ and $c \not\preceq b$.
5. But by the principle of *indiscernibility of identicals* it follows from 2 and 3 that $c \preceq a$ and $c \preceq b$.

Opponents of the innocence thesis blame 2 for this contradiction.
But their interpretation of 'the whole is identical with its parts' is $c = a$ *and* $c = b$ instead of $c = (a$ *and* $b)$.

Unrestricted Composition: Identification

How to understand '$c = (a \textit{ and } b)$'?

Take, e.g., 'Tris and Iseult are human beings.' We paraphrase it by 'Tris is a human being and Iseult is a human being.' (primary: '*and*')

That would be the opponents interpretation.

Take, e.g., 'Tris and Iseult are in love.' We paraphrase it the same way as above or as 'Tris is in love with Iseult.' (primary: 'is in love')

Analogue we have to take the 'is identical with' in the innocence thesis as primary: '$c = (a \textit{ and } b)$' is to be understood as $c = d$ whereupon 'd' refers exactly to a and b.

Thesis (OII)

$sum(a, b) = d$ whereupon 'd' refers to a and b.

Identification Approach

Four Theories on Predication

Recall our thesis OII: $sum(a, b) = d$ wherupon 'd' refers to a and b.

As one can see, the specific signs in the thesis are '*sum*', '=' and 'refers to ... and ...'. So we need at least three theories:

- one on plural reference, which will be done with theories on plural predication (ε)
- one on identity
- and one on composition, which will be done with mereology

Let us start with predication!

First Theory on Predication

Our first theory on predication is a theory on singular predication (that is: only the copula 'is' occurs in predicational statements).

More precisely: It is the common formal theory of predication:

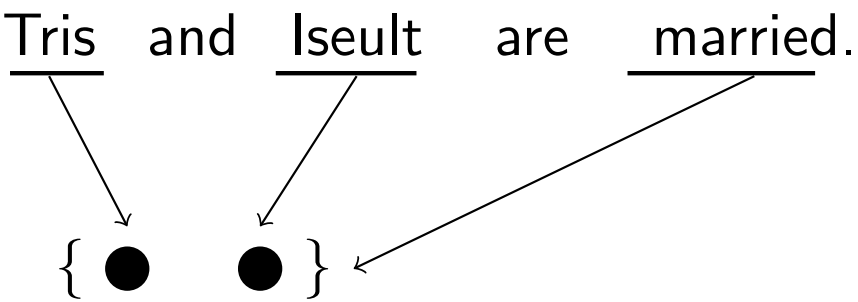
- $\forall x \forall y(x \varepsilon y \leftrightarrow x \in y)$, where for $\in$ hold all axioms of set theory

To say that Iseult is beautiful is according to AP1 modelled by saying that Iseult is a member of the set of all beautiful entities.

All 'are'-predications within ordinary language are translated into 'is'-predications:

'Tris and Iseult are beautiful.' $\Rightarrow$ 'Tris is beautiful and Iseult is beautiful.'

First Theory on Predication (AP1): ε



Second Theory on Predication

The following theories on predication are theories on plural predication (that is: also the copula 'are' occurs in predicational statements).

More precisely: It is a slightly modified version of Leśniewskis theory of predication (cf. Betti 2010):

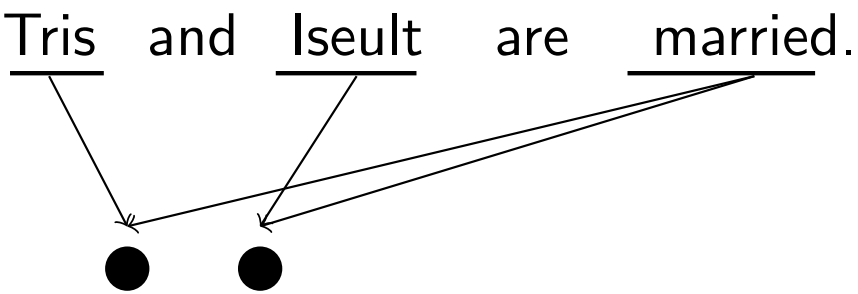
- AP2 $\forall x \exists y y \varepsilon x$ &
  $\forall x \forall y (x \varepsilon y \leftrightarrow \forall z_1 \forall z_2 (z_1 \varepsilon x \,\&\, z_2 \varepsilon x \rightarrow z_1 \varepsilon z_2) \,\&\, \forall z (z \varepsilon x \rightarrow z \varepsilon y))$

To say that Iseult is beautiful is according to AP2 modelled by saying that (i) there is an entity that is Iseult, (ii) that every entities that are Iseult are each other and (iii) that every entity that is Iseult is one of the entities that are beautiful.

Claims about non-existent entities or predicational statements about more than one entity are false:

'Tris and Iseult are beautiful.' $\Rightarrow$ 'Tris is one of the beautiful and Iseult is one of the beautiful.'

Second Theory on Predication (AP2): ε



Third Theory on Predication

A modified version of AP2: We detach the uniqueness condition:

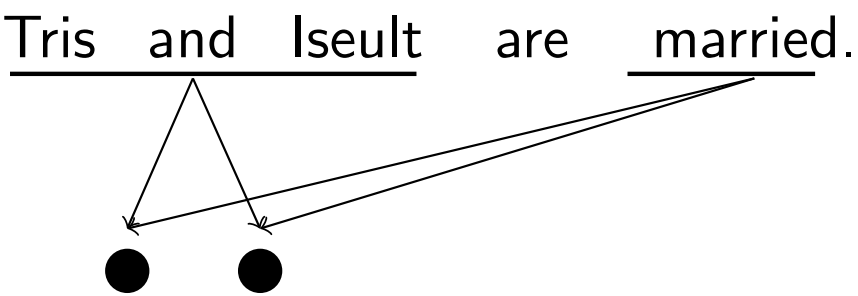
- AP3 $\forall x \exists y y \varepsilon x \ \& \ \forall x \forall y (x \varepsilon y \leftrightarrow \forall z (z \varepsilon x \rightarrow z \varepsilon y))$

To say that Iseult is beautiful is according to AP3 modelled by stating that (i) there is an entity that is Iseult and (ii) that every entity that is Iseult is also one of the entities that are beautiful.

Claims about non-existent entities are false. Predicational statements about more than one entity are possible:

'Tris and Iseult are beautiful.' remains unchanged.

Third Theory on Predication (AP3): ε



Fourth Theory on Predication

Again a modification: we weaken the inclusion condition:

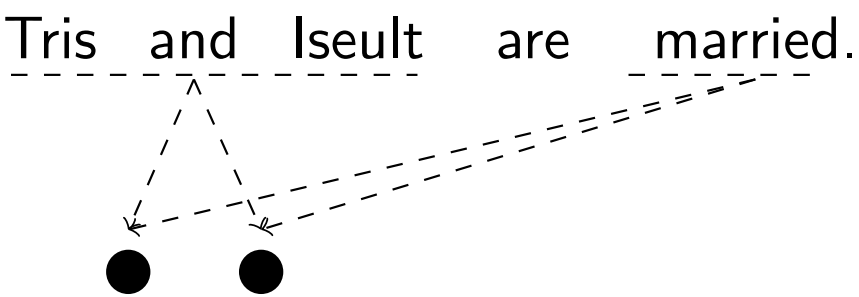
- AP4 $\forall x \exists y y \varepsilon x \;\&\; \forall x \forall y (x \varepsilon y \leftrightarrow \exists z (z \varepsilon x \;\&\; z \varepsilon y))$

To say that Iseult is beautiful is according to AP4 modelled by stating that (i) there is an entity that is Iseult and (ii) that some of the entities that are Iseult are also some of the entities that are beautiful.

Claims about non-existent entities are false. Predicational statements about more than one entity are possible, but a little bit *gavagaian*. . .

'Tris and Iseult are beautiful.' holds even if one of them is not beautiful.

Fourth Theory on Predication (AP4): ε



Theory of Identity

All four theories are strong enough to build up a theory of identity just by definition:

- DI1 $x = y \leftrightarrow \forall z(z \varepsilon x \leftrightarrow z \varepsilon y)$

Where for $=$ holds reflexivity and also the principle of the *indiscernibility of identicals*:

- T1 $\forall x \forall y(x = y \rightarrow (Q(x) \leftrightarrow Q(y)))$ (Leibniz' Law I)

For AP3 and AP4 also a variant of the principle of the *identity of indiscernibles* holds:

- T2 $\forall x \forall y(\forall z(x \varepsilon z \leftrightarrow y \varepsilon z) \rightarrow x = y)$ (Leibniz' Law II)

Theory on Composition

They (ε-theories) are also strong enough to build up a theory of composition (mereology) just with the help of one non-conservative axiom:

- (M) $x \preceq y \leftrightarrow \forall z(z\varepsilon x \rightarrow z\varepsilon y)$
- (M) $x \prec y \leftrightarrow x \preceq y \;\&\; x \neq y$
- (M) $x \circ y \leftrightarrow \exists z(z\varepsilon x \;\&\; z\varepsilon y)$
- (M) $A(x) \leftrightarrow \neg\exists y y\varepsilon x$
- (M) $\forall x \forall y(sum(x,y) = \iota z \forall z_1(z_1 \circ z \leftrightarrow (z_1 \circ x \vee z_1 \circ y)))$

The non-conservative part of mereology:

- (T3) $\forall x \forall y \exists z \forall z_1(z_1 \circ z \leftrightarrow (z_1 \circ x \vee z_1 \circ y))$

(Unrestricted Composition)

Recall: We are investigating a thesis in favour of exactly this principle of *unrestricted composition*, the innocence thesis.

By the way: the uniqueness condition for composition is conservative.

Results on OII

Now we have all ingredients for investigating OII, namely theories on: ε, $=$ and *sum*.

We have asked whether mereology is ontologically innocent or not.

Opponents just sayed unqualified: no!

We conclude: well, it depends...

...it depends on your theory of predication:

- T4 AP1 (plus identity theory plus mereology) contradicts OII.
- T5 AP2 (plus ...) is compatible with OII.
- T6 AP3 (plus ...) is compatible with OII.
- T7 AP4 (plus ...) entails OII.

Results on OII

There is even a more interesting result:

- T8 OII, mereology (*sum*) and the theory of identity ($=$) entail AP4 (ε).

So, to claim that mereological abstraction is logical abstraction within the identification approach presented here is to be a *gavagaian* philosopher of language.

Things are different if we handle plural predication by plural quantification!

Plural Predication as Plural Quantification

One can construct mereology in MSL (monadic second order logic) ...

- Universal Instantiation:
  $\forall X^1 \varphi[X^1] \rightarrow \varphi[X^1/P^1]$
- Comprehension:
  $\exists X^1 \forall x(X^1(x) \leftrightarrow \varphi[x])$ (where X^1 is not free in φ)

... purely by definitions:

- $X \sqsubseteq Y \leftrightarrow \forall x(X(x) \rightarrow Y(x))$
- $X \bullet Y \leftrightarrow \exists Z(Z \sqsubseteq X \& Z \sqsubseteq Y)$
- $\Sigma x \varphi[x] \equiv X \leftrightarrow \forall Y(X \bullet Y \leftrightarrow \exists Z(Z \bullet Y \wedge \forall x(Z(x) \leftrightarrow \varphi[x])))$

The – in FOL non-conservative – existential claim for the composition operation is a consequence of the comprehension axiom in MSL.

Plural Predication as Plural Quantification

Nice features of such a construction:

- The construction is managable (no pairing function is definable in the construction)
- There are, from a nominalistic point of view, acceptable paraphrases for the expressions of MSL ('x is one of the Xs', 'all Xs are Ys' etc. – (cf. Boolos 1984))
- The construction also works, if one restricts MSL – e.g.: $\exists x \varphi[x] \rightarrow \exists X^1 \forall x (X^1(x) \leftrightarrow \varphi[x]))$

Conclusion

Conclusion

We have explicated a common approach to ontological innocence: the identification approach with plural predication.

But the identification approach succeeds only in presupposing a gavagaian theory of plural predication.

So this approach is not very promising with respect to logical abstraction.

But there seem to be some promising ways out of the problem – namely plural quantification. . .

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