Concept Formation and Reduction by Analogies

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Introduction

We are familiar with comparisons like:

- Electric current in a conductor is like water in a pipe
- Memes are like genes
- Interaction with God is like triangulation

Such comparisons are often seen as some kind of concept formation by analogies.

But what does it mean to form/construct concepts by analogies and what are they good for?

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Concept Formation by Analogies

Relevance of Analogies

Analogies are no hot-topic in philosophy of science.

To get nevertheless involved into recent debates I will try to embed results about analogies into modern theoretical frames:

- Conclusions by analogy: frame of confirmation theory (cf. Hesse 1964)
- Concept formation by analogy: frame of reductionism

A First Characterization of Analogies

Analogies are frequently used in scientific descriptions and explanations.

Indicators for analogical reasoning and descriptions are comparing phrases:

- 'similar as'
- 'likewise'
- 'analogically'
- etc.

Let's begin with a short overview of the purposes of analogical reasoning and descriptions!

The Purpose of Analogical Usage of Language

There are four main purposes of analogical usage of language (cf.: (Bunge 1973), (Hempel 1970), (Bocheński 1959) and (Weingartner 1976)):

- Abbreviation: e.g., in mathematics analogies are used for abbreviating proofs (cf. 'without limiting the generality').
- Didactics: illustration of claims. E.g.:
 - Claims about a unknown domain G₁ (space-time-curvature by heavy masses)
 - well-known domain G₂
 (masses on elastic surfaces)

This mode of describing is very frequent in teaching and presenting scientific theories.

The Purpose of Analogical Usage of Language

domain of inquiry." (cf. Hempel 1970, p. 438)

- Context of discovery: the value of analogies is to be found in the heuristics for finding new regularities; Carl G. Hempel, e.g., claims:

 "In order to appraise the explanatory significance of analogical models, and more generally of analogies based on nomic isomorphism, let us suppose that some "new" field of inquiry is being explored, and that we try to explain the phenomena encountered in it by analogical reference to some "old", previously explored
- Context of justification; E.g.:
 - Someone wants to argue that a claim A_1 is a consequence of a theory T_1 , but has no exact theoretical frame, proof etc. for this claim.
 - Then she may show that there are very relevant relations of analogy between T_1 and another (well-known) theory T_2 and between A_1 and a consequence of T_2 , call it A_2 ($T_2 \vdash A_2$).
 - By establishing these analogies she then may claim: $T_1 \sim A_1$.

The Purpose of Analogical Usage of Language

F1–F3 suggest that analogical usage of language is principally (i.e. without consideration of psychological facts of restricted imagination power, hypotheses invention, demonstration power etc.) redundant in science.

Different investigations of F4 come to different results about the value of analogical usage of language: there are two opposing parties.

One group (of philosophers of science) does *not* accept analogies for the justification of theories. E.g.: Pierre Maurice Marie Duhem in (Duhem 1998) and Hempel in (Hempel 1970, chpt.6).

Whereas another group accepts analogies for the justification of theories. E.g.: Joseph M. Bocheński in (Bocheński 1959).

A similar difference of opinions seems to hold also for concept formation by analogies (e.g.: is the meme-gene-analogy acceptable for speaking of meme theories?).

Comparison of water in a pipe with current in a conductor:

Shortened analogical description:

'Electric current in a conductor is like water in a pipe.'

Take, e.g., the law of Hagen-Poiseulle and Ohm's law:

(V...volume of fluid, c...speed, p_i ...pressure)

- $v_1 v_2 = \frac{1}{k}$ (1... amperage, k... conductance, v_i ... potential)

It is well known that c varies indirect proportional with the length of the pipe:

$$c \sim \frac{1}{l_1}$$

$$(I_1...$$
 length of the pipe)

Analogical to this fact it holds that k varies indirect proportional with the length of the conductor:

$$k \sim \frac{1}{l_2}$$

$$(I_2...length of the conductor)$$

Furthermore it holds that:

$$V \sim r_1^4$$

$$(r_1 \dots radius of the pipe)$$

But it holds (not similarly) that:

$$I \sim r_2^2$$

 $(r_2...$ radius of the conductor)

Analogical usage of language about two different domains (e.g., physics of liquids and electromagnetism) means that some regularity descriptions are syntactically isomorph, that is: $V \mapsto I$, $c \mapsto k$, $p_i \mapsto v_i$ and vice versa. With the help of this example the main problem of analogical usage of language is easily expressed:

Which descriptions of regularities within one domain of investigation are adequately adoptable for descriptions of regularities within another domain of investigation?

The simplest solution to the problem would be a restrictive definition (cf. Hempel 1970, p.434):

Instead of 'x is analogue to y' one defines 'x is analogue to y w.r.t. L_i '.

According to this solution it holds: V is analogue to I with respect to L1 and L2, but not with respect to L5 and L6.

Let is be a (partial) mapping (on the vocabulary of both theories):

- is(I) = V
- $is(v_i) = p_i$
- is(k) = c
- $is(l_2) = l_1$

Then one may generalize is inductively:

- For all ...: $is(P^n(t_1,...,t_n)) = is(P^n)(is(t_1),...,is(t_n))$
- For all terms t_1, t_2 : $is(t_1 \equiv t_2) = is(t_1) \equiv is(t_2)$
- For all formulas A: $is(\neg A) = \neg is(A)$
- For all formulas A, B: is(A&B) = is(A)&is(B)
- For all formulas A and variables x: $is(\forall xA) = \forall xis(A)$

And describe the analogical relations by: L1 \Rightarrow is(L1), L3 \Rightarrow is(L3)

Concept Formation by Analogies

What does it mean that by these analogical relations current (I) and conductance k are in some way characterized?

The analogical relations can be restated logically equivalent as:

- L1⇒ (*is*(L1) ⇔L1)
- L3⇒ (*is*(L3) ⇔L3)

Which may be seen as conditionalized contextual definitions of:

I, k, v_i and l_2

Perhaps by such restatements one can make some sense of 'concept formation by analogies'.

Main problems:

- conditionalized multiple characterization of an expression
- difference between contextual definitions and non-definitional axioms

Reductionism

A Preliminary Distinction

Within the discussion of reductionism one should distinguish more or less sharply between the methods of (cf. Moulines 2008, p.79):

- Elimination
- Reduction
- Definition

Some examples for a first characterization:

• Elimination of some theoretical terms by the method of Ramsey Frustration aggression theory (cf. Dollard et al. 1970): $T = \{ \forall x (Frus(x) \rightarrow Aggr(x)), \forall x (SOff(x) \rightarrow Frus(x)), \forall x (Aggr(x) \rightarrow (Shou(x) \lor Hitt(x) \lor \dots)) \}$ $T^R = \exists P \exists Q (\forall x (P(x) \rightarrow Q(x)) \land \forall x (SOff(x) \rightarrow P(x)) \land \forall x (Q(x) \rightarrow (Shou(x) \lor Hitt(x) \lor \dots)))$

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\textit{Result: } T^R \textit{ without a specific theoretical vocabular and } \textit{empCont}(T) = \textit{empCont}(\{T^R\})
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- Reduction of Thermodynamics to Statistical Mechanics (cf. the discussion in Nagel 1961/1979, chpt. 11)
- Definition of 'is an ordered pair' by the method of Wiener/Kuratowski In the following we are going to talk about reductions and definitions, but not about elimination in a broad sense.

Different Kinds of Reductionism

There are different positions subsumed under the label 'reductionism'. One may categorize the most important positions in the following way (similar to Crane 2000):

- Ontological reduction: That is to identify all objects of the domain of one theory with some objects of the domain of another theory.
- Translational reduction:
 - Term-by-term translation
 - Sentence-by-sentence translation
 - Law-by-law translation
 - Theory-by-theory translation (mostly called 'explanational reduction')

Once again we make a restriction: we will only consider translational reduction and stick mainly to term-by-term translations.

NB: There are also more general distinctions of reductionism (cf. the introduction in Charles 1992) and (Cat 2007):

normative (ideal of science) vs. descriptive (real development of science)

Different Kinds of Translational Reductions

Given this categorization, the main task to do is to clarify what is meant by 'translation'.

Classical answers to this problem are as follows:

- **1** An expression y is translatable to a set of expressions X iff y is definable with the help of X.
- **1** An expression y is translatable to a set of expressions X iff y is partially definable with the help of X.
- **1** An expression y is translatable to a set of expressions X iff y is connectable to elements of X by so-called rules of correspondence.

:

Relative to these clarifications there hold different relations between the different kinds of translational reduction.

E.g., in case of interpreting 'translation' in the sense of R1.1:

 $R1 \Rightarrow R2$, $R2 \Rightarrow R3$, $R2 \Rightarrow R4$, $R3 \not\Rightarrow R4$, ...

Carnap's Three Phases of Reductionism

To give an example for different kinds of reductions and reductionism we will have a short look at some work done in this area by Carnap:

- 1928: In his "Aufbau" (Carnap 1928/2003) as well as in (Carnap 1931) and (Carnap 1932) Carnap argues for R1.1.
- 1936: In "Testability and Meaning" he provides a framework for R1.3.
- 1963: In the replies in his Paul Arthur Schilpp-collection he argues for a weakened version of R2 in the context of physicalism:
 - Reduction: A sentence is physicalistically reducable iff it is empirical confirm- and disconfirmable.
 - Reductionism: All sentences of science are mathematical sentences or physicalistically reducable (to be understood as normative statement).

There seems to be a strong connection between translational reductions and definitional criteria. So let's have—after presenting briefly a similar distinction by Ernest Nagel—a look on some of the later ones!

Nagel's View of Reductionism in a Nutshell

There is also a similar distinction in (Nagel 1961/1979, chpt. 11 and p.338): "Reduction [...] is the explanation of a theory or a set of experimental laws established in one area of inquiry, by a theory usually though not invariably formulated for some other domain."

Nagel proposes two conditions for a reduction of, e.g., ψ to ϕ (cf. Nagel 1961/1979, pp.353f):

- ① Condition of derivability: the laws of ψ can be derived of the laws of ϕ (incl. some coordinating definitions).
- 2 Condition of connectability: If there is a theoretical expression of ψ which is no such expression of ϕ , then one needs to provide some principles for connecting this expression with those of ϕ .

A principle of connection as refered to in 2 could be (cf. Nagel 1961/1979, p.354):

- ① Explication
- 2 Definition
- 3 Any usual connection (e.g. by a bilateral reduction sentence)

Classical Definitional Criteria

Non-Creativity (Lukasiewicz 1970):

Definition

 T_2 is a non-creative extension of T_1 iff for all $\phi \in \mathcal{L}_{T_1}$ it holds that: $\vdash^{T_2} \phi$ iff $\vdash^{T_1} \phi$.

That is:
$$\{\phi : \vdash^{T_2} \phi\} \cap \mathcal{L}_{T_1} = \{\phi : \vdash^{T_1} \phi\}$$

Example:

An extension of Peano arithmetics by a partial definition of a sign for division is a non-creative extension (of Peano arithmetics).

Classical Definitional Criteria

Eliminability (Ajdukiewicz 1958):

Definition

t of T_2 is eliminiable with respect to T_1 iff for all $\phi \in \mathcal{L}_{T_1,t}$ there is a $\psi \in \mathcal{L}_{T_1}$, such that: $\vdash^{T_2} (\phi \leftrightarrow \psi)$.

Example:

'Prime' (of mathematics) is eliminable with respect to Peano arithmetics. A sign for division (of mathematics) is not eliminable with respect to Peano arithmetics.

The Relevance of these Criteria

They are highly relevant for translations and they are independent (vs. the adequacy of the analysis in (Kleinknecht 1981)).

The ideal of translation: Let α be an ideal cognitive agent, that is: (α is logically omniscient) and $\alpha's$ operation of understanding expressions is that of logical interpretation.

Further let α understand the following signs in the following way:

- $\varphi_{\alpha}(P_1^1) = \{1, 2\}, \ \varphi_{\alpha}(P_2^1) = \{1\}, \ \varphi_{\alpha}(c_1) = 1, \ \varphi_{\alpha}(c_2) = 2$
- Hence: $\varphi_{\alpha}(P_1^1(c_1)) = \varphi_{\alpha}(P_1^1(c_2)) = \varphi_{\alpha}(P_2^1(c_1)) = \mathcal{T}$, $\varphi_{\alpha}(P_2^1(c_2)) = \mathcal{F}$, etc.

Now assume that α learns a new sign P_3^1 (i.e. $\varphi_{\alpha} \mapsto \varphi'_{\alpha}/\varphi''_{\alpha}$):

Directive 1:
$$\varphi'_{\alpha}((P_3^1(a_1) \leftrightarrow (P_1^1(a_1) \land \neg a_1 \equiv c_2))) = \mathcal{T}$$

Directive 2:
$$\varphi''_{\alpha}(((P_3^1(a_1) \leftrightarrow (P_1^1(a_1) \land \neg a_1 \equiv c_2) \land P_2^1(c_2))) = \mathcal{T}$$

According to both directives it holds: $\varphi'_{\alpha}(P_3^1) = \varphi''_{\alpha}(P_3^1) = \{1\}.$

But only according to directive 1 a translation of, e.g., $P_3^1(x)$ by, e.g., $P_2^1(x)$ is correct as far as $\varphi'_{\alpha}(P_2^1) = \varphi_{\alpha}(P_2^1)$, but $\varphi''_{\alpha}(P_2^1) \neq \varphi_{\alpha}(P_2^1)$.

The Relevance of these Criteria

There is a nice theorem about these criteria regarding reductionism:

Recall! According to R1.1 it holds:

Definition (Strong term-by-term reduction)

An expression t of T_2 is reducable to a set of expressions of T_1 iff T_2 is an extension of T_1 by a definition of t.

Example: Is '... is aggressive' of ψ reducible to a set of expressions of ϕ ?

Theorem ((cf. Kutschera 1967, chpt.6.3))

 T_2 is an extension of T_1 by a definition of t iff T_2 is a non-creative extension of T_1 and t of T_2 is eliminable with respect to T_1 .

So, definitions are the one and only directives guaranteeing translatability in the most narrow sense.

(The—with respect to reductionism—relevant direction is \Leftarrow)

The Relevance of these Criteria

There are some other nice relevant features of these criteria:

- ☑ Preservation of consistency among theory extension
- Preservation of completeness among theory extension
- Preservation of translatability among theory extension

Nevertheless, the requirement for translatability in the most narrow sense is too strong for reductionism—e.g., T_2 is reducible to T_1 in the most narrow sense of translatability only, if there is a $T_3 \subseteq T_1$ such that T_2 and T_3 are synonymous (cf. for this notion Kanger 1968).

So, one has to weaken this requirement.

Some Weaker Criteria for Translations

This is, e.g., done in R1.2 by requiring only partial definability.

And this is to give up the requirement of eliminability for translations.

- Preservation of consistency among theory extension
- ☑ Preservation of completeness among theory extension (indicated in (Ebbinghaus 1969, p.39) and (Suppes 1957, §8.6))
- ☑ Preservation of translatability among theory extension

Our example about the frustration aggression theory can be reconstructed as a reduction of this kind:

```
Recall the example of the frustration aggression theory: T = \{ \forall x (Frus(x) \rightarrow Aggr(x)), \\ \forall x (SOff(x) \rightarrow Frus(x)), \ \forall x (Aggr(x) \rightarrow (Shou(x) \lor Hitt(x) \lor \dots)) \} The last two axioms can be logically equivalent reformulated as partial definitions: T' = \{ \forall x (Frus(x) \rightarrow Aggr(x)), \ \forall x (SOff(x) \rightarrow (Frus(x) \leftrightarrow SOff(x))), \ \forall x (\neg (Shou(x) \lor Hitt(x) \lor \dots)) \} \dots) \rightarrow (Aggr(x) \leftrightarrow (Shou(x) \lor Hitt(x) \lor \dots)) \}
```

Some Weaker Criteria for Translations

But even requiring R1.2 is too strong. Suppose Aggr to be a dispositional expression, then the given reduction is inadequate. Again, one has to weaken the requirement as, e.g., done in R1.3 by requiring only connectivity by so-called rules of correspondence.

And this is to give up even the requirement of non-creativity for translations, as far as, e.g., bilateral reduction sentences are creative:

```
The reduction of Aggr as dispositional expression \forall x \forall t (SOff(x,t) \rightarrow (Aggr(x) \leftrightarrow (Shou(x,t) \lor Hitt(x,t) \lor \dots))) leads exactly to the following additional suppositions about tests (cf. Essler 1975): \forall x (\exists t (SOff(x,t) \land (Shou(x,t) \lor Hitt(x,t) \lor \dots)) \rightarrow \forall t (SOff(x,t) \rightarrow (Shou(x,t) \lor Hitt(x,t) \lor \dots)))
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And this results in a loss of all mentioned features.

- Preservation of consistency among theory extension
- Preservation of completeness among theory extension
- ☑ Preservation of translatability among theory extension

Further Problems for Classical Reductionism

Although weakend in such a way and loosing some important formal features, there are still some serious problems left.

E.g., one of the main objections against physicalism ($\phi \Rightarrow \psi$) are the following ones (cf. Beckermann 2001, p.90):

- Mental predicates are cluster concepts— there are no sufficient and necessary conditions for defining them physicalistically.
- If one tries to define them, then one produces a circle—at least in describing test-reaction-pairs.
- Mental predicates are at the best only partially definable.

As far as 2 seems to be discussable only with respect to single cases, and as far as 3 seems to be addressed at least partly by reductions weaker than R1.1, we are going to concentrate only on 1.

The Problem of Finding Adequate Conditions

The objection against classical reductionism in 1 is justified by the claim that—to give an example— $SOff(c_1)$ sometimes leads to $Shou(c_1)$ or $Hitt(c_1)$ or ..., but not always, and that because of this such reductions are inadequate (cf. Beckermann 2001, pp.87f).

We may demonstrate this objection by the given example of the R1.3-reduction of the frustration aggression theory:

In detail, the argument runs against the supposition about tests made within R1.3-reductions:

```
\forall x (\exists t (SOff(x,t) \land (Shou(x,t) \lor Hitt(x,t) \lor \dots)) \rightarrow \forall t (SOff(x,t) \rightarrow (Shou(x,t) \lor Hitt(x,t) \lor \dots)))
```

The most natural way to address this objection seems to try to overcome this problem by weakening this supposition about tests:

```
\forall x (\exists t (SOff(x,t) \land (Shou(x,t) \lor Hitt(x,t) \lor \dots)) \rightarrow usually it holds for t(SOff(x,t) \rightarrow (Shou(x,t) \lor Hitt(x,t) \lor \dots))), etc.
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Non-Classical Reductionism

Such a weakening corresponds to a weakening of the requirements for R1.3-reductions.

One may try, e.g.:

```
Usually it holds for x and t(SOff(x,t) \rightarrow (Aggr(x) \leftrightarrow (Shou(x,t) \lor Hitt(x,t) \lor \dots)))
```

And this is to allow not only reductions within classical logic, but also within non-classical logic:

Definition (Non-classical term-by-term reduction)

An expression t of T_2 is reducable to a set of expressions of T_1 iff t of T_2 is non-classically connectable via so-called rules of correspondence with expressions of T_1 .

Different Kinds of R4-Reductions

Different kinds of theory-by-theory reductions (explanational reductions):

- **a** The strictest forms of reductions are derivations: $T_1 \vdash T_2$
- **3** A more moderate form of a reduction is definitional derivation. Let D be a set of definitions of some concepts of T_2 with the help of concepts of T_1 . Then reduction in this sense is a demonstration of $T_1 \cup D \vdash T_2$.
- A yet more moderate form of reductionism is definitional and reductional derivation. Let D be a set as described above and R be a set of reduction sentences interrelating some concepts of T₂ with concepts of T₁ (e.g., by meaning postulates, bilateral reduction sentences etc.). Then reduction in this sense is a demonstration of T₁ ∪ D ∪ R ⊢ T₂.

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An Extension of Non-Classical Reductionism

If the meaning of 'concept formation by analogies' is clarified in the indicated way, one may try to provide reductions by analogies.

(A research programme could be, e.g., the reduction of meme theories to gene theories by the help of analogies.)

Let *D* and *R* be as described above and let *A* be a set of concept formations of some concepts of T_2 by analogies with respect to the concepts of T_1 . Then analogical reduction is a demonstration of $T_1 ∪ D ∪ R ∪ A ⊢ T_2$.

One may also try to find out which criteria of theory reduction/extension are satisfied by such reductions/extensions (of course not: eliminability etc.)

Summary

Summary

- There are two ways of using analogies in science:
 - Conclusions by analogies
 - Concept formation by analogies
- Needed: An explication of 'concept fromation by analogies'. (wip)
- One possible solution: partial contextual definitions further investigations about formal properties are needed. (wip)
- Relevant modern context for conclusions by analogies: Bayesianism.
- Relevant modern context for concept formation by analogies: Reductionism.
 - Possible applications: Supervenience thesis (the meme-gene-analogy is in support of the claim that cultural evolution supervenes biological evolution) etc.

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